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Gravity and antigravity in a brane world with metastable gravitons.

Comment on: G. Dvali, G. Gabadadze and M. Porrati, "Metastable gravitons and infinite volume extra dimensions," hep-th/0002190
and
C. Csaki, J. Erlich, T. J. Hollowood, "Graviton propagators, brane bending and bending of light in theories with quasi-localized gravity," hep-th/0003020

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Abstract

In the framework of a five-dimensional three-brane model with quasi-localized gravitons we evaluate metric perturbations induced on the positive tension brane by matter residing thereon. We find that at intermediate distances, the effective four-dimensional theory coincides, up to small corrections, with General Relativity. This is in accord with Csaki, Erlich and Hollowood and in contrast to Dvali, Gabadadze and Porrati. We show, however, that at ultra-large distances this effective four-dimensional theory becomes dramatically different: conventional tensor gravity changes into scalar anti-gravity.

The papers by Dvali, Gabadadze and Porrati, [1], and Csaki, Erlich and Hollowood, [2], address the issue of whether four-dimensional gravity is phenomenologically acceptable in a class of brane models with infinite extra dimensions in which the five-dimensional gravitons have a metastable "bound state", rather than a genuine zero mode. A model of this sort has been proposed in Refs. [3,4] and is a variation of the Randall–Sundrum (RS) scenario for a non-compact fifth dimension [5]. The construction with metastable gravitons has been put in a more general setting in Refs. [6,1]. It has been argued in Ref. [1] that models with metastable gravitons are not viable: from the four-dimensional point of view, gravitons are effectively massive and hence appear to suffer from a van Dam–Veltman–Zakharov [7] discontinuity in the propagator in the massless limit. In particular, it has been claimed [1] that the prediction for the deflection of light by massive bodies would be considerably different from that of General Relativity. The issue has recently been analyzed in more detail in Ref. [2], where explicit calculations of four dimensional gravity have been performed along the lines of Garriga and Tanaka [8], and Giddings, Katz and Randall [9]. The outcome of that analysis is that four-dimensional gravity has been claimed to be in fact Einsteinian, despite the peculiarity of apparently massive gravitons.

In this comment we also apply the Garriga–Tanaka (GT) technique to obtain effective four-dimensional gravity at the linearized level, considering as an example the model of Refs. [3,4]. We find that at intermediate distances (which should extend from microscopic to very large scales in a phenomenologically acceptable model) four dimensional gravity is indeed Einsteinian, in accord with Ref. [2] and in contrast to Ref. [1]. However, at ultra-large scales we find a new phenomenon: four-dimensional gravity changes dramatically, becoming *scalar anti-gravity* rather than tensor gravity. This may or may not signal an internal inconsistency of the models under discussion.

To recapitulate, the set up of Refs. [3,4] is as follows. The model has five dimensions and contains one brane with tension $\sigma > 0$ and two branes with equal tensions $-\sigma/2$ placed at equal distances to the right and to the left of the positive tension brane in the fifth direction. There is a reflection symmetry, $z \to -z$, which enables one to consider explicitly only the region to the right of the positive tension brane (hereafter z denotes the fifth coordinate). Conventional matter resides on the central positive tension brane. The bulk cosmological constant between the branes, Λ, is negative, whereas it is equal to zero to the right of the negative tension brane. With appropriately tuned Λ, there exists a solution to the five-dimensional Einstein equations for which both positive and negative tension branes are at rest at $z = 0$ and $z = z_c$ respectively, z_c being an arbitrary constant. The metric of this solution is

$$ds^2 = a^2(z)\eta_{\mu\nu}dx^\mu dx^\nu - dz^2 \tag{1}$$

where

$$a(z) = \begin{cases} e^{-kz} & 0 < z < z_c \\ e^{-kz_c} \equiv a_- & z > z_c \end{cases} \tag{2}$$

The constant k is related to σ and Λ. The four-dimensional hypersurfaces $z = const.$ are flat, the five-dimensional space-time is flat to the right of the negative-tension brane and anti-de Sitter between the branes. The spacetime to the left of the positive tension brane is a mirror image of this set-up.

This background has two different length scales, k^{-1} and

$$r_c = k^{-1} e^{3kz_c} \tag{3}$$

These are assumed to be well separated, $r_c \gg k^{-1}$. It has been argued in Ref. [4] that the extra dimension "opens up" both at short distances, $r \ll k^{-1}$ and ultra-long ones, $r \gg r_c$.

To find the four-dimensional gravity experienced by matter residing on the positive tension brane, we follow GT and consider a Gaussian-Normal (GN) gauge

$$g_{zz} = -1 \qquad g_{z\mu} = 0 \tag{4}$$

In the bulk, one can further restrict the gauge to be transverse-tracefree (TTF)

$$h^\mu_\mu = h^\mu_{\nu,\mu} = 0 \tag{5}$$

Hereafter $h_{\mu\nu}$ are metric perturbarions; indices are raised and lowered by the four-dimensional Minkowski metric. The linearized Einstein equations in the bulk take one and the same simple form for all components of $h_{\mu\nu}$,

$$\begin{cases} h'' - 4k^2 h - \frac{1}{a^2}\Box^{(4)} h = 0 & 0 < z < z_c \\ h'' - \frac{1}{a_-^2}\Box^{(4)} h = 0 & z > z_c \end{cases} \tag{6}$$

It is convenient, however, to formulate the junction conditions on the positive tension brane in the local GN frame. In this frame, metric perturbations $\bar{h}_{\mu\nu}$ are not transverse-tracefree, so the two sets of perturbations are related in the bulk between the two branes by a five-dimensional gauge transformation preserving (4),

$$\bar{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{k}\hat{\xi}^5_{,\mu\nu} - 2ka^2\eta_{\mu\nu}\hat{\xi}^5 + a^2(\xi_{\mu,\nu} + \xi_{\nu,\mu}) \tag{7}$$

where $\xi_\mu(x)$ and $\hat{\xi}^5(x)$ are the gauge parameters. Notice that if ξ^5 is not zero, there is a 'shift' in the location of the wall relative to an observer at infinity, i.e. the wall appears bent to such an observer (as discussed in [8,9]). Physically, this simply represents the fact that the wall GN frame is constructed by integrating normal geodesics from the wall, and in the presence of matter these geodesics will be distorted, thereby altering the proper distance between the wall and infinity. In fact, one finds a similar "bending" of the equatorial plane in the Schwarzschild spacetime if one tries to impose a local GN frame away from the horizon.

In the presence of additional matter on the positive tension brane with energy momentum $T_{\mu\nu}$, the junction conditions on this brane read

$$\bar{h}'_{\mu\nu} + 2k\bar{h}_{\mu\nu} = 8\pi G_5 \left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu} T^\lambda_\lambda \right) \tag{8}$$

where G_5 is the five-dimensional gravitational constant. The solution to equations (5) – (8) has been obtained by Garriga and Tanaka. They found that $\hat{\xi}^5$ obeys

$$\Box^{(4)}\hat{\xi}^5 = -\frac{4\pi}{3} G_5 T^\lambda_\lambda \tag{9}$$

We will need the expression for the induced metric on the positive tension brane. Up to terms that can be gauged away on this brane, the induced metric is [8]

$$\bar{h}_{\mu\nu}(z=0) = h^{(m)}_{\mu\nu} - 2k\eta_{\mu\nu}\hat{\xi}^5 \tag{10}$$

where

$$h^{(m)}_{\mu\nu} = 16\pi G_5 \int dx' \; G^{(5)}_R(x,x';z=z'=0)\left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}T^\lambda_\lambda\right)(x') \tag{11}$$

Here $G^{(5)}_R$ is the retarded Green's function of eq. (6) with appropriate (source-free) junction conditions on the two branes. This Green's function is mirror-symmetric and obeys

$$\left[\partial_z^2 - 4k^2\theta(z_c - z) - \frac{1}{a^2}\Box^{(4)} + 4k\delta(z) - 2k\delta(z-z_c)\right] G^{(5)}_R(x,x';z,z') = \delta(x-x')\delta(z-z') \tag{12}$$

Let us consider the case of the static source first. It has been found in Ref. [4] that for $k^{-1} \ll r \ll r_c$, the leading behavior of the static Green's function (given by $\int \; dt \; G^{(5)}_R(z = z' = 0)$) is the same as in the RS model (up to small corrections), and corresponds to a $1/r$ potential. Hence, at intermediate distances the analysis is identical to GT, and the induced metric is the same as in the linearized four-dimensional General Relativity. This is in accord with Ref. [2].

On the other hand, it follows from Ref. [4] that at ultra-large distances, $r \gg r_c$, the contribution (11) behaves like $1/r^2$ (the fifth dimension "opens up"). There remains, however, the second term in eq.(10). Since eq.(9) has a four-dimensional form, this term gives rise to a $1/r$ potential (missed in Ref. [4]) even at ultra-large distances. For a point-like static source of unit mass, the corresponding gravitational potential is

$$V(r) \equiv \frac{1}{2}\bar{h}_{00}(r) = +\frac{1}{3}G_4\frac{1}{r} \tag{13}$$

where $G_4 = kG_5$ is the four-dimensional Newton's constant entering also into the conventional Newton's law at intermediate distances. We see that at $r \gg r_c$, four-dimensional gravity is induced by the trace of energy-momentum tensor and has a repulsive $1/r$ potential. At ultra-large distances tensor gravity changes to scalar anti-gravity.

Likewise, the four-dimensional gravitational waves emitted by non-static sources are conventional tensor ones at intermediate distances and transform into scalar waves at ultra-large distances (the relevant distance scale being different from r_c due to relativistic effects, see Ref. [4]). Indeed, the first term in eq. (10) dissipates [4], whereas the second term survives, again due to the four-dimensional structure of eq. (9).

These two cases illustrate the general property of eq. (10): the first term becomes irrelevant at ultra-large distances (the physical reason being the metastability of the five-dimensional graviton bound state), so the four-dimensional gravity (in effect, anti-gravity) is entirely due to the second, scalar term.

This bizarre feature of models with a metastable graviton bound state obviously deserves further investigation. In particular, it will be interesting to identify the four-dimensional massless scalar mode, which is present at ultra-large distances, among the free sourceless

perturbations. This mode is unlikely to be the radion [10,11], studied in this model in Ref. [3]: the radion would show up at intermediate distances, as well as at ultra-large ones*; furthermore, the experience [13] with models where the distance between the branes is stabilized [14] suggests that the massless four-dimensional mode parametrized by $\hat{\xi}^5$ exists even if the radion is made massive.

More importantly, one would like to understand whether anti-gravity at ultra-large distances is a signal of an intrinsic inconsistency of this class of models, or simply a signal that physics is intrinsically five-dimensional at these scales. In four dimensions, scalar antigravity requires either negative kinetic and gradient energy or a ghost. Whether or not a similar feature is inherent in models with extra dimensions remains an open question. If it is, there still would remain a possibility that fields with negative energy might be acceptable, as their effect might show up at ultra-large distances only.

We note finally, that anti-gravity may not be a special feature of models with quasi-localized gravitons. It is also possible that this phenomenon may be present in models of the type suggested by Kogan et. al. [15], where some Kaluza–Klein graviton excitations are extremely light. The same question about internal consistency then would apply to these models as well.

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*The radion presumably couples exponentially weakly to the matter on the positive tension brane, as it does [8,3] in the two-brane model of Randall and Sundrum [12]. This may be the reason why the radion effects have not been revealed by the analyses made in Ref. [2] and this note.

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